Sectoral Heterogeneity, Income Inequality and Productivity Dynamics

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Abstract

An intricate dynamic pattern has been commonly observed in many developed countries during the past decades. This pattern contains a simultaneous rise in the following economic variables: (i) total factor productivity, (ii) educated labor supply, (iii) wage-gap between high- and low-skilled workers, and (iv) income inequality. Typical explanations for the different elements of this pattern assume a skill-biased technical change (SBTC) or capital-skill complementarity. In this study we offer a complementing explanation for these phenomena, which is based on sectoral heterogeneity and endogenous factor mobility, rather than on an SBTC. We show that sectoral heterogeneity can amplify the effects of a technical change, whether skill-biased or general, in a manner that generates the four elements of the above described dynamic pattern. Furthermore, inequality can perform also a Kuznets-curve pattern, as was observed in several countries, in contrast to the inequality dynamics in typical SBTC models.

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1. Introduction

In the past few decades, most developed economies have experienced a dynamic pattern of physical and human capital accumulation, rising income inequality, rising wage-gap between skilled and unskilled workers, and rising total factor productivity.[1] The literature on economic growth has usually explained these dynamics by assuming a skill biased technical change (SBTC). In this study we emphasize the role of sectoral heterogeneity as a complementing explanation for the observed dynamics of TFP, skill supply, wage-gap and inequality.

Differences between sectors, and the evolution of these difference, are an important feature of the observed growth, at least since the industrial revolution. For example, even prior to the Industrial Revolution, industrial production took place, and the service sector existed too. Yet, they were much smaller than the agricultural sector, unlike their relative contribution to GDP nowadays, with the growth in these two sectors playing a key role in the industrial revolution. A more recent example is the share of the service sector out of total value added in the USA, which rose from 0.6 in the 1950s to 0.8 in 2000 (Buera and Kaboski, 2012).

In this study we analyze a case in which growth springs from investments in physical and human capital, in a setting in which there are two production sectors, one more advanced than the other. As a result, along the growth process there is a gradual increase in both:

- the share of the physical capital that firms invest in the more advanced production sectors of the economy, out of the total investments of these firms,
- the amount of people who choose to invest in their education in order to work in these advanced sectors.

[1]For a more detailed description of these trends, see Galor & Moav (2000), Krusell et al. (2000), Sauer et al. (2015), and Chusseau et. al (2008) and Nolan et. al (2019) for two surveys of the literature on the rising inequality, wage premium and their relation to skill-biased technical change.

Each of these two processes offers a positive feedback for the other, as within the advanced sector physical capital and skilled workers are complements.

This mechanism needs not be perceived necessarily as an alternative to the SBTC explanation, but rather as a complementary one, where the sectoral heterogeneity propagates the effect of the SBTC. Thus, it is possible to view growth as a process where skill-biased technological changes are infrequent, and whenever such a change occurs in a relatively advanced sector, it ignites a process where more physical and human capital is directed to that sector in subsequent periods. A prominent historical example for that is the contribution of Watt's improvement of the steam engine to the industrialization in subsequent decades. For several decades after the introduction of Watt's engine in the 1860s, much of its effect on growth was based on the massively increasing adoption of such engines, rather than on additional major improvements of it.[2] And, alongside the acquiring of these engines, firms were acquiring more technically skilled workers for the operation and maintenance of these engines and the machines connected to them.[3] More recently, and in a rather similar manner, after the invention of the personal computer, and its connection to the World Wide Web, much of the resulting growth was due to the spreading use of computers, rather than from major improvements to computers. In this case too, the acquiring of computers by firms, also led to hiring more workers with computer skills, and this rising demand led to increased human capital investments which are focused on acquiring computers skills.

We present this mechanism via a general equilibrium dynamic model with two sectors: one more productive than the other. In order to work in the more productive sector, an individual has to acquire education. Acquiring education is an individual choice based on

[2] In fact, there were many improvements to the steam engine in the decades after Watt has presented his engine in the 1860s. Yet, none of them can be viewed as a major change until, at about 1800, as Watt' patent rights expired, Trevithick has invented the high-pressure engine and immediately based the first railway locomotive on it. See Nuvolari, et al. (2006) for further details.

[3] See for example Franck and Galor (2019) and De Pleijt et. al (2019), who show how the adoption of steam engines has increased the investment in education in France and in England, respectively.

expected future wages and on the cost of education. Firms choose their technology endogenously, by choosing between operating in the more advanced sector or in the less advanced one. The analysis of the resulting macroeconomic equilibrium highlights a feedback relationship between investment in education and investment in physical capital in the advanced sector, as one promotes the marginal productivity of the other.

We find that the equilibrium dynamics are characterized by a monotonic increase in output, human capital and in the relative size of the advanced sector. In addition, the wage gap between high-skilled and low-skilled workers rises over time. This springs from the feedback mechanism described above, which dominates the negative effect that the rising supply of high-skilled workers exerts on this wage gap.

We show that the rising wage gap may lead to a dynamic pattern of a rising income inequality. However, it is also possible that income inequality shall not be monotonically rising, but instead, shall exhibit Kuznets curve dynamics, in which it initially rises, and from a certain point in time it may decline, even though the wage gap continues to rise. This decline may occur when the number of high-skilled workers becomes sufficiently large to make the relative equality among the high-skilled workers dominate the rising gap between the average wages of the two groups of workers. This is a different mechanism from the one which generates the Kuznets curve dynamics in most models of the relevant literature, with Acemoglu (1998) and Galor and Moav (2000) as two prominent examples. Usually, in these models a skill-biased shock raises inequality by raising the wage-gap between skilled and unskilled workers, with a subsequent increase in the supply of educated labor, which lowers the education premium and inequality. Thus, these mechanisms cannot generate falling inequality alongside a continuously rising wage gap.

The increasing share of physical capital allocated to the advanced sector, and the rise in the share of the population that chooses to become high-skilled, make TFP rise over time. This rise in TFP is not an outcome of a technical change (as technology is assumed constant in this model), but rather due to an endogenous shift of investments in physical and human capital from the less productive sector to the advanced one, which increases the contribution of the TFP of the advanced sector to the economy-wide TFP.[4] Hence, our paper highlights an indirect channel through which investment in human capital promotes economic growth. The direct channel is based on the result that having more human capital raises production for any given allocation of physical capital in each sector. The indirect channel is based on the property that having more human capital also attracts a larger share of the economy's physical capital to the more advanced sector.

Our paper relates to the growing literature that analyzes structural change and its relation to economic growth and also to income inequality.[5] (See, for example, Baymul & Sen, 2020; Buera & Kaboski, 2012; Buera et al., 2015; Comin et al., 2018; Diao et al., 2017, Duarte and Restuccia, 2010; Haraguchi et al., 2017, Herrendorf et al., 2014, McMillan et al., 2014, Trew, 2014). Some of these studies are theoretical and the others are empirical, documenting the two processes of growth and structural change. The theoretical ones (e.g. Buera & Kaboski, 2012; Duarte & Restuccia, 2010) differ from our model in several aspects: First the models that they present usually assume homogenous workers, and thus free movement between sectors. As such, they cannot be related to the rise in the skill premium or income inequality. Second, they strongly rely on non-homothetic preferences, which with an exogenous increase in income, yield a non-balanced increase in demand for different products.[6] Our model, in contrast, is agnostic to the type of product, and focuses on the endogenous supply of production factors as

[4] This result is close in its nature to that of Zeira (2009). In his model, an increase of the stock of educated workers increases the profitability of adopting a new type of machines, and thus promotes economic growth indirectly.
[5] The literature on the rise of the service sector is vast, and sometimes referred to as “Unbalanced Growth”, starting with Baumol (1967), and many others in the past few decades, e.g., Kongsamut et al., (2001) or Acemoglu and Guerrieri (2008). For more details on this literature, see Acemoglu (2008, Ch. 20), and Buera &Kaboski (2012)
[6] Buera et al. (2015) also provide evidence that GDP per capita grew alongside with a shift in the production from low-skill intensive sectors to high-skill intensive ones, in the USA during the 20th century. They also show that this process occurred hand in hand with a rise in the skill premium.

a source for changes over time in the relative sizes of the different sectors.[7]

The article is organized as follows: Section 2 presents the model and finds its equilibrium dynamics; Section 3 presents the main results about the dynamics patterns of TFP, wage-gap, inequality and relatives sizes of the sectors, and also presents a comparative statics analysis of the steady state; Section 4 offers some concluding remarks.

2. The Model

Consider a closed OLG economy with a constant population along time. Each generation lives three periods. In the first period of her or his life each individual chooses whether to acquire higher education or not; In the second period of life, all individuals work according to their educational level, consume, save, and each one of them gives birth to one offspring; In the third period of life all individuals are retirees, and consume all their savings.

There are two production sectors, one more advanced than the other. In order to work in the advanced sector, individuals have to acquire education, which is costly; Firms too, have to decide in which sector to invest. Markets are fully competitive, so factor prices equal their marginal product.

2.1. Production and Factor Prices

Aggregate output at period t satisfies:

(1) $$Y_t = A_H \cdot \left(K_t^H\right)^{\alpha} \cdot H_t^{1-\alpha} + A_L \cdot \left(K_t^L\right)^{\alpha} \cdot L_t^{1-\alpha} = A_H \cdot H_t \cdot \left(k_t^H\right)^{\alpha} + A_L \cdot L_t \cdot \left(k_t^L\right)^{\alpha}$$

[7] Swiecky (2017) highlights the role of sector-biased technical change as main driving force behind structural change. Thus, he also shows the importance of supply side mechanisms in structural change. Nevertheless, in our model, this change appears even without a sector specific technical change.

where $A^H > A^L$ are sector specific technology parameters, K_t^O is the capital employed in sector $O \in \{H, L\}$ at period t; H_t and L_t are the stocks of high-skilled and low-skilled labor that are employed in production respectively; and $k_t^O \equiv K_t^O/O_t$.[8]

Factor markets are competitive, and therefore factor prices equal their marginal product:

(2) $$R_t = \frac{\alpha \cdot A_H}{\left(k_t^H\right)^{1-\alpha}} = \frac{\alpha \cdot A_L}{\left(k_t^L\right)^{1-\alpha}},$$

and

(3) $$w_t^O = (1-\alpha) \cdot A_O \cdot \left(k_t^O\right)^{\alpha},$$

where R_t is the rental rate of physical capital and w_t^O is the period t of a worker in sector O.

2.2 Individuals

All individuals live three periods. In the first life period each one can acquire, at choice, higher education, which determines if he will be a high-skilled or a low-skilled worker. In the second period he inelastically supplies his time unit to the labor market, consumes, gives birth to one offspring, and saves for his retirement period consumption. Specifically, each individual i that is born at period t-1 maximizes the following utility function:

[8] We follow Galor & Zeira (1993) in assuming two alternative production processes for the same product, where the sectors differ in their skill intensity. This assumption is merely a simplifying one, and an alternative modelling where the two production processes produce different goods, could be used here as well, leading to the same qualitative result. Such an alternative modelling would significantly complicate the analysis as it requires the modelling of preferences and demand for each good, and using the resulting equilibrium price ratio for creating a measure of aggregate production. The main results of this study will emerge under this alternative modelling as well because it preserves the two main forces underlying these results: (1) A difference between sectors in sectoral total productivity (under a differentiated goods modelling – measured in terms of the numerator good); (2) The need for individuals to investment in their human capital in order to work in the advanced sector.

(4) $$U\left(c_t^i, c_{t+1}^i\right) = (1-\beta)\cdot\ln\left(c_t^i\right) + \beta\cdot\ln\left(c_{t+1}^i\right)$$

under the budget constraint:

(5) $$c_t^i + \frac{c_{t+1}^i}{R_{t+1}} \leq W_t^i$$

where $\beta \in (0,1)$, and c_t^i is the consumption of individual *i* at period *t*, and W_t^i is the wealth of individual *i* at period *t*. This immediately leads to $c_t^i = (1-\beta)\cdot W_t^i$ and $c_{t+1}^i = \frac{s_t^i}{R_{t+1}} = \beta\cdot W_t^i$.

Each individual *i* that acquires higher education must pay the cost h^i at the first period of his life. Thus:

(6) $$W_t^i = \begin{cases} w_t^L & \text{if } i \text{ is a low-skilled worker} \\ w_t^H - h^i\cdot R_t & \text{if } i \text{ is a skilled worker} \end{cases}$$

By (4), each individual *i*, born at $t-1$, acquires education only if it raises W_t^i, and by (6) this takes place if and only if h^i is below the following threshold:

(7) $$\bar{h}_t \equiv \frac{w_{t+1}^H - w_{t+1}^L}{R_{t+1}}.$$

We assume that there is heterogeneity between individuals in their return to education. There are two main reasons for this assumption: First, it is a very plausible assumption; Second, in order to study how sectoral heterogeneity affects income inequality and wage differences between skilled and unskilled workers, we must have this heterogeneity, otherwise in equilibrium all individuals are indifferent about acquiring education or not. In this case, changes over time in inequality would require assuming that the technology parameters or the education cost change over time. This diverges the analysis from its focus on how sectoral heterogeneity affects the macroeconomic equilibrium, and also complicates the analysis significantly. There exist two main approaches in the literature to model this heterogeneity. One approach is to model the return to education in terms of efficiency units, suggesting that the more abled can produce more in a given unit of time (e.g., Galor & Tsiddon, 1997). The second approach assumes that the more abled have a lower cost of acquiring education. This can take the form of a direct cost (for example, less investment in private tutors) or as an opportunity cost (less time spent in studying and thus more time for work or leisure), e.g. Maoz and Moav (1999). Both approaches lead to the same result: there is a single individual who is indifferent between acquiring education or not, and all individuals with more ability than this individual acquire education. For simplicity of the analysis we take the latter approach.

Specifically, we assume that $h^i \sim U(0, 1)$ and i.i.d. across generations. Thus, $\bar{h}_t$ of the individuals that were born at period t acquire education, while $1-\bar{h}_t$ of them don't, implying that the period $t + 1$ supplies of skilled and unskilled labor are $H_{t+1} = \bar{h}_t$ and $L_{t+1} = 1-\bar{h}_t$.

Note from (2) that:

$$k_t^H = (1+\gamma)\cdot k_t^L, \tag{8}$$

where $\gamma \equiv \left(\frac{A_H}{A_L}\right)^{\frac{1}{1-\alpha}} - 1 > 0$, and the inequality follows from $A_H > A_L$. Equation (8) implies that in equilibrium, the ratio of capital per worker between the two sectors rises with the ratio of productivities between the two sectors.

2.3 The Dynamical System

Applying (2) and (3) in (7) and then applying it in $H_{t+1} = \bar{h}_t$ yields:

(9) $$k_{t+1}^{L} = \frac{\alpha}{(1-\alpha)\cdot\gamma} \cdot H_{t+1}$$

Applying (8), (9) and $L_t = 1 - H_t$ in (1) yields:

(10) $$Y_t = A_L \cdot \left[\frac{\alpha}{(1-\alpha)\cdot\gamma}\right]^{\alpha} \cdot H_t^{\alpha} \cdot (\gamma \cdot H_t + 1).$$

The physical and human capital for period *t*+1 are formed during period *t*, and satisfy:

(11) $$K_{t+1}^{H} + K_{t+1}^{L} + \tfrac{1}{2} \cdot \bar{h}_t^{\,2} = \beta \cdot \left[(1-\alpha)\cdot Y_t - \tfrac{1}{2} \cdot R_t \cdot \bar{h}_{t-1}^{\,2}\right],$$

where the LHS of (11) shows the three types of investment, based on the period *t* expenditures on investment being $\int_0^{\bar{h}_t} h^i \cdot f(h^i) \cdot dh^i = \tfrac{1}{2} \cdot \bar{h}_2^{\,2}$, and the RHS is period *t* individual savings based on their aggregate wages being $(1-\alpha)\cdot Y_t$, and their debt from period $t-1$ being $\tfrac{1}{2} \cdot \bar{h}_{t-1}^{\,2}$.

Applying (2), (3), (7), and (10) in (11) and rearranging terms yields the following autonomous first-order dynamic equation:

(12) $$H_{t+1} = \frac{-\alpha + \sqrt{\alpha^2 + \phi \cdot \left(\gamma \cdot H_t^{1+\alpha} + 2 \cdot H_t^{\alpha}\right)}}{(1+\alpha)\cdot\gamma} \equiv f(H_t),$$

where:

$$(13) \qquad \phi \equiv \alpha^{\alpha} \cdot (1+\alpha) \cdot (1-\alpha)^{2-\alpha} \cdot \gamma^{2-\alpha} \cdot A_L \cdot \beta > 0 .$$

It is straightforward to notice from (12) and (13) that $f(0)=0$ and $f'(H_t)>0$ for all $H_t>0$, and that $\lim_{H_t \to 0} f'(H_t)=\infty$ and $\lim_{H_t \to \infty} f'(H_t)=0$. These properties imply that the dynamical system $H_{t+1}=f(H_t)$ has at least one non-trivial steady state equilibrium with $H > 0$. This steady state is unique (as we prove in the Appendix), which implies that this steady state is also stable and that the economy is approaching it monotonically.

3. Main results

After finding the equilibrium path of the economy in the previous section, we now turn to examine the evolution of TFP dynamics, wage-gap and income inequality, and sectors relative sizes, along this path. Throughout this analysis we are going to assume the initial conditions locate the economy at period 0, to the left of the steady state level of H_t, so the evolution of the economy is characterized by output growth, rather than by a declining output. We conclude this section with a comparative statics analysis of the steady state of the economy.

3.1 TFP Dynamics

A well-known fact is that countries with a higher human capital endowment tend to have a higher level of TFP. Explanations for this phenomenon have varied from cross-country technological differences (e.g. Romer, 1993, and Caselli and Coleman, 2006), to explanations based social infrastructure differences (Hall, 1999) or barriers to physical capital (Restuccia, 2004). Acemoglu & Zilibotti (2001) argue that in the steady state, countries with different human capital endowments have different productivities, because of a mismatch between machines and human capital. Their paper, however, relies on the premise that human capital is exogenous and constant over time. The following proposition shows that the same mechanism in which human capital attracts investment in physical capital yields differences in TFP. Yet, since human capital is endogenous in our model, these differences exist only during the transitional dynamics.

Proposition 1: Along the development course of the economy, Total Factor Productivity is monotonically increasing.

Proof: In our model, TFP is given by:

$$(14) \qquad TFP_t = \frac{Y_t}{K_t^{\alpha} \cdot \left(\frac{A_H}{A_L} \cdot H_t + L_t\right)^{1-\alpha}},$$

where K_t is the total amount of capital in the economy in period t. Applying (10) for Y_t and $K_t^H = k_t^H \cdot H_t$, $K_t^L = k_t^L \cdot (1 - H_t)$, (8) and (9) for K_t, and rearranging yields:

$$(15) \qquad TFP_t = \left[A_L \cdot \frac{\gamma \cdot H_t + 1}{(A_H - A_L) \cdot H_t + 1}\right]^{1-\alpha}$$

Differentiating (15) yields that $\frac{dTFP_t}{dH_t} > 0$. Thus, since H_t rises over time, so does TFP. □

3.2 Wage Gap and Inequality Dynamics

It is a well-known fact that in the last few decades many economies have experienced both a rise in the wage gap and a rise in income inequality accompanied with a rise in the educated labor force. The two main explanations for the coincidence of these three phenomena were SBTC (Galor & Moav, 2000, Acemoglu, 1998) and capital-skill complementarity (Krusell et al., 2000). In this section we provide another possible explanation for these phenomena. In particular, we show that along the transitional dynamics the wage gap increases, and that income inequality increases in the beginning of the development process, but may fall later, as in a Kuznets curve pattern.

From (3), (8) and $\gamma \equiv \left(\frac{A_H}{A_L}\right)^{\frac{1}{1-\alpha}} - 1$ it follows that:

(16) $$w_t^H - w_t^L = (1-\alpha)\cdot\gamma\cdot A_L\cdot\left(k_t^L\right)^{\alpha}$$

Thus, by (9), as H_t rises over time, so does k_t^L , which increases the education wage-gap.

Note that the wage gap increases despite the rise of the high-skilled labor force and the decline of the low-skilled labor force. This is induced by the larger physical capital investments in the advanced sector, relative to the less-advanced one, as implied by (8).

The result about the wage gap assists us to explore the dynamics of income inequality in the economy. The following proposition shows that income inequality increases for small values of H_t and declines for sufficiently high values of H_t. Since the dynamics of the economy are characterized by a monotonous rise in H_t, income inequality rises at the outset of development, and may decline at later stages of development. Hence, income inequality may increase along time, or exhibit a Kuznets curve pattern.

Proposition 2: At the beginning of the development process, income inequality increases as H_t increases. At later stages of development, income inequality may decline as H_t increases.

Proof: The average income at period *t* is given by:

(17) $$\overline{w}_t = H_t \cdot w_t^H + (1 - H_t) \cdot w_t^L$$

We measure income inequality by the variance of income which is given by:

(18) $$\sigma_t^2 = H_t \cdot \left(w_t^H - \overline{w}_t\right)^2 + (1 - H_t) \cdot \left(w_t^L - \overline{w}_t\right)^2 = (1 - H_t) \cdot H_t \cdot \left(w_t^H - w_t^L\right)^2$$

$$= \left[(1-\alpha)^{1-\alpha} \cdot \gamma^{1-\alpha} \cdot \alpha^\alpha \cdot A_L\right]^2 \cdot (1 - H_t) \cdot H_t^{1+2\cdot\alpha},$$

where the second equality follows from (9) and (16). From (18) it follows that:

(19) $$\frac{d\sigma_t^2}{dH_t} = \left[(1-\alpha)^{1-\alpha} \cdot \gamma^{1-\alpha} \cdot A_L \cdot \alpha^\alpha\right]^2 \cdot H_t^{2\cdot\alpha} \cdot \left[1 + 2\cdot\alpha - 2\cdot(1+\alpha)\cdot H_t\right].$$

The derivative is positive as long as $0 < H_t < \frac{1+2\cdot\alpha}{2+2\cdot\alpha}$, and negative as long as $\frac{1+2\cdot\alpha}{2+2\cdot\alpha} < H_t < 1$. Since H_t increases along time, the variance in income increases at the outset of the development process, and may decline at later stages of the development process, if H_t passes $\frac{1+2\cdot\alpha}{2+2\cdot\alpha}$.

The same result, via a similar proof, is obtained by using the Gini coefficient and not the variance of income. □

Among the two possible dynamic patterns that the proposition establishes, the one of monotonically rising inequality is the one shared in recent decades by most countries, and in particular OECD countries. A recent important example for the second pattern of a Kuznets curve is Germany's income inequality, which after a long period of a stable increase, has been gradually declining since 2010 (Hutter, 2017).[9]

Proposition 2 sheds light on the dynamics of income inequality in the economy. As the economy develops, two forces with opposite signs affect income inequality: wage inequality between the two groups of workers (the wage gap) and the relative abundance of high-skilled workers. As the economy develops the wage gap increases, a force that increases inequality, but the relative abundance of high-skilled workers increases as well, which in turn decreases income inequality. According to Proposition 2, at the outset of the development process, the former is greater than the latter, while in later stages of development the opposite may occur. Note that this result is not a trivial outcome of the fact that the factor $(1-H_t)\cdot H_t$ has an inverse-U shape, as occurs in models where wages are exogenous. Here, as (18) shows, this factor is multiplied by the wage-gap which falls at an infinite rate at the vicinity of $H_t = 0$, due to the endogenous determination of wages and the Inada condition of the production functions. In a single sector model this fall in the wage-gap dominates the rise in $H_t \cdot (1-H_t)$ in that vicinity.[10]

3.3 Sectors Relative Size

In examining the relative sizes of the sectors, and the dynamics of these relative sizes, we look at the *share* of each sector out of the total production, physical capital, and employment.

As for employment, the analysis in sub-section 2.3 has shown that the dynamics of the economy are characterized by a monotonic increase in the number of workers in the advanced sector, H_t, and a decline in the number of workers in the less productive sector, $L_t = 1 - H_t$.

We continue with the share of the advanced sector's output out of aggregate output, which we define and calculate as follows:

[9] Baymul & Sen (2020) provide empirical evidence of the effect of structural change on inequality and find evidence for dynamic patterns of inequality consistent with those described by Proposition 2 in this study.
[10] See, for example, Maoz & Moav (1999), where in a model of a single sector, inequality is monotonically falling for all positive H_t, as one example of many for the dominance of the wage effect induced by the Inada conditions.

(20) $$s_t^{Y,H} \equiv \frac{A_H \cdot \left(K_t^H\right)^{\alpha} \cdot H_t^{1-\alpha}}{Y_t} = \frac{\gamma \cdot H_t}{\gamma \cdot H_t + 1 - H_t},$$

where the second equality follows from (1) and (8). Differentiating (20) yields:

(21) $$s_t^{Y,H}{}'(H_t) = \frac{\gamma}{(\gamma \cdot H_t + 1 - H_t)^2} > 0.$$

As shown in section 2.3, along the development course of the economy, H_t is monotonically rising. This, taken together with (21), implies that over time, as the economy grows, the share of the production of the advanced sector out of total production is rising over time too. These dynamics are consistent with the findings of Buera et al. (2015).

In contrast, the share of the production of the less productive sector out of total production is falling along the development course of the economy. Specifically, we denote this share by $s_t^{Y,L}$ and define and calculate it similarly to $s_t^{Y,H}$ which immediately leads to $s_t^{Y.L} = 1 - s_t^{Y,H}$ and to:

(22) $$s_t^{Y,L}{}'(H_t) = -s_t^{Y,H}{}'(H_t) < 0.$$

Examining the share of the physical capital in each sector out of the total physical capital in the economy is done via a similar analysis, and yields similar results. Specifically, we define and calculate this ratio by:

(23) $$s_t^{K,H} \equiv \frac{K_t^H}{K_t^H + K_t^L} = \frac{H_t \cdot (1+\gamma)}{H_t \cdot \gamma + 1},$$

where the second equality follows from the definitions of k_t^H and k_t^L, and from (8). Straightforward differentiation of (23) yields:

$$(24) \qquad s_t^{K,H}{}'(H_t) = \frac{1+\gamma}{(H_t \cdot \gamma + 1)^2} > 0,$$

which implies that along the development course of the economy, the share of the physical capital in the advanced sector out of total physical capital is rising over time. Likewise, the share of the physical capital in the less productive sector out of total physical capital, which we denote by $s_t^{K,L}$, is falling along the development course of the economy.

3.4 Steady State Analysis

In section 2.3 we proved that the dynamical system converges to a unique steady state, with H_t rising over time. In this section we analyze how changes in the parameters of the model, A_H, A_L and β, affect the steady state values of different variables in the model. We provide an analytical proof for the effect of β, and demonstrate the effects of the two productivity parameters via simulations of the model. An analytical proof for the effects of the productivity parameters includes tedious math (available from the authors upon request).

3.4.1. The effect of a change in β

From (12) it follows that a higher level of β yields a higher level of H_{t+1} for each $H_t > 0$. This implies that a higher level of β also leads to a higher level of H_t in the steady state. This, taken together with (8), (9), and (10), implies that a higher level of β also leads to a higher steady state levels of H, Y, k_t^H, k_t^L, w_t^H, w_t^L, $s_t^{Y,H}$ and $s_t^{K,H}$. The intuition behind the effect that an increase in β has on most of these variables is straightforward: a higher β leads to more savings and therefore to greater investments in human and physical capital. A less intuitive result is the positive effect on $s_t^{Y,H}$ and $s_t^{K,H}$ which occurs although the increased tendency for savings due to a higher β is a general one, directed towards any profitable investment. Yet, the advanced sector benefits more than the less productive sector from a higher β because its operation requires also investment in human capital, alongside the investment in physical capital.

3.4.2 The effect of A_H

Figure 1 presents the steady state values of the number of skilled workers (*H*), total output (*Y*), total factor productivity, and inequality (as measured by the variance of incomes), obtained by simulating the model for different values for A_H. As can be seen in the figure, higher values of A_H generate also higher steady state levels of *H*, *Y* and TFP. The intuition behind this result is based on two effects: First, a *direct effect* which implies that for any given combination of production factors in each sector, a higher level of A_H leads to higher output, and therefore savings and investments.

The second effect of a change in A_H, which we call *incentive effect*, implies that a higher A_H creates a higher incentive to direct a larger part of the savings towards investments in the advanced sector, both in human and in physical capital. These two forces work in the same direction, leading to the results shown in the figure.

In the case of inequality, the pattern is different: as Figure 1 shows, the steady state level of inequality is an inverse u-shape function of the level of A_H. The intuition behind this result is similar to the one explained in section 3.2: income inequality in the economy is comprised from a wage gap between the two types of workers, and the size of each group of workers. When A_H is low enough, the higher steady state wage gap associated with a higher A_H is the dominant effect and leads to higher inequality. The opposite occurs when A_H is sufficiently high and the steady state size of the skilled workers group is high enough so that the relative equality within that group dominates the effect of the wage gap.

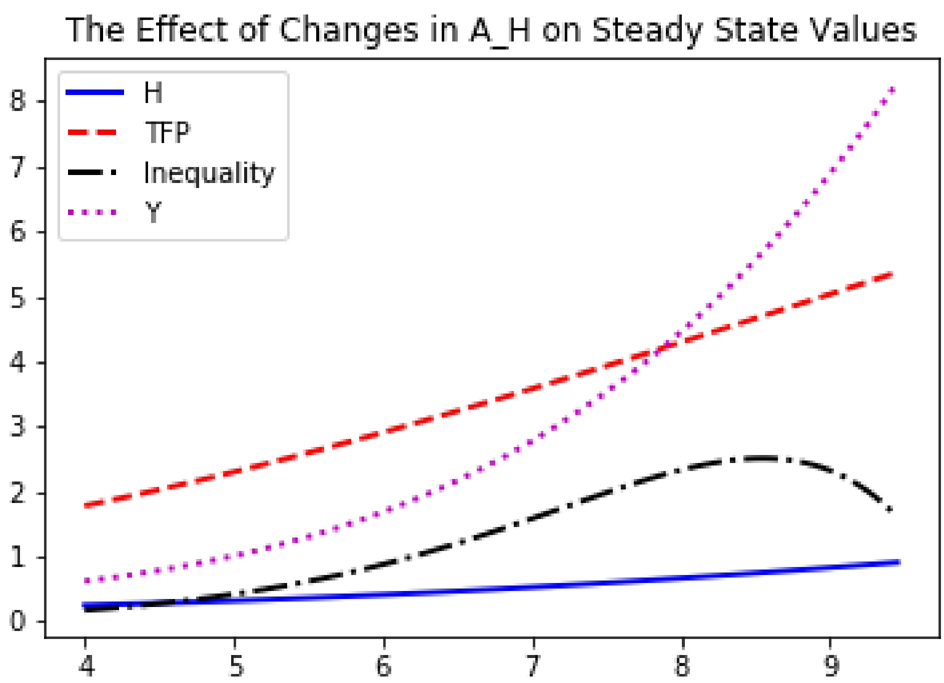

Figure 1: steady state values of Human capital, output, TFP an inequality as function of A_H. Parameter values: $A_L = 1$, $\alpha = 0.5$, $\beta = 0.6$.

3.3.3 The Effect of A_L

Figure 2 shows how different values of A_L affect the steady state values of H, Y, TFP, and inequality.

The two forces described in the previous section, the *direct effect* and the *incentive effect*, are in the focus of the analysis in this case too. Yet, in contrast to the case of a change in A_H, here these two effects work in contradicting directions with regard to the investments in the advanced sector and their steady state consequences. In particular, the figure shows that with regard to H, the *incentive effect* is dominant for all values of A_L and that the steady state level of H decreases with A_L throughout the entire relevant range. As for Y and TFP, the figure shows alternating dominancy between the two effects and therefore a u-shaped connection between A_L and the steady state levels of Y and TFP. The figure also shows a u-shaped connection between inequality and A_L which follows from the negative connection between A_L and H and the manner by which H affects inequality, as described in sub-section 3.2.

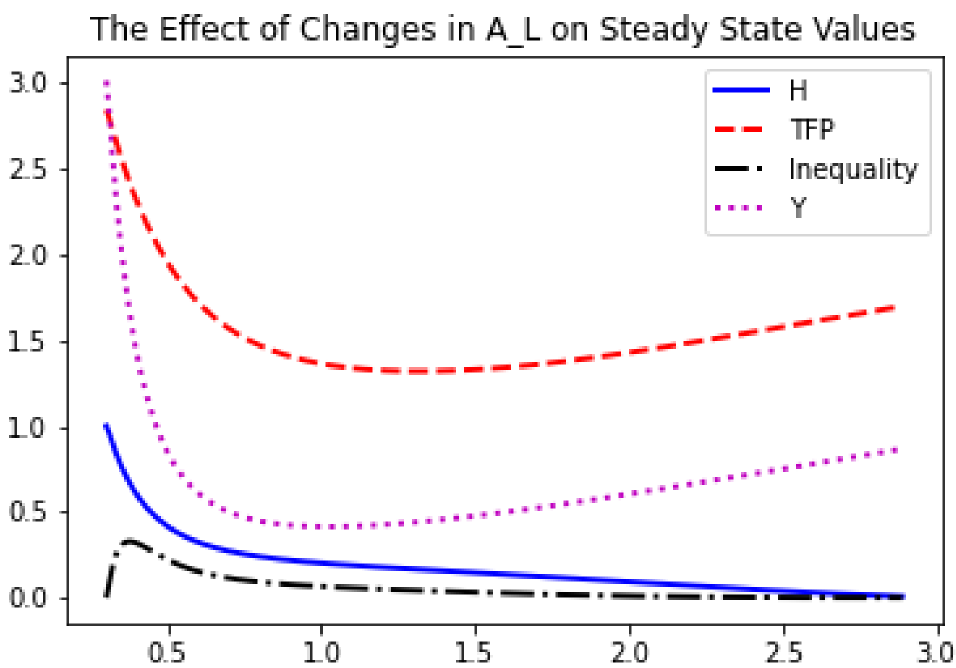

Figure 2: steady state values of Human capital, output, TFP an inequality as function of A_L. Parameter values: $A_H = 3$, $\alpha = 0.5$, $\beta = 0.6$.

4. Conclusions

We have presented a general equilibrium dynamic model with two sectors: one more productive than the other. We used this model to analyze how the heterogeneity in sector productivity affects the dynamics of physical and human capital accumulation, income inequality and total factor productivity.

The analysis of the resulting macroeconomic equilibrium highlights a feedback relationship between investment in education and investment of physical capital in the advanced sector, as one promotes the marginal productivity of the other. This feedback mechanism leads to an increase in the wage gap over time. We also find that the rising wage gap may lead to a dynamic pattern of a rising income inequality. However, it is also possible that income inequality shall not be monotonically rising, but instead, shall exhibit Kuznets curve dynamics, in which it initially rises and from a certain point in time begins to decline, even though the wage-gap continues to rise. The decline in inequality occurs when the number of high-skilled workers is sufficiently large to make the relative equality within the high-skilled workers dominate the rising inequality between the two groups of workers.

This feedback mechanism also highlighted another indirect channel through which human capital promotes economic growth. As human capital is accumulated, more physical capital is allocated to the more advanced sector, and thus output grows faster. This effect lead to the rise in the TFP over time. We showed that in this sense our results are close in their nature to Acemoglu & Zilibotti (2001), Caselli & Coleman (2006) and Zeira (2009). Yet, we analyze the transitional dynamics and not merely the steady state equilibrium. We also added to their results and used the model for analyzing inequality pattern. These results about the rise in the wage gap, income inequality and productivity, fit the empirical findings presented by a massive body of literature.

Our results also imply that a SBTC, which can materialize in our model by an increase in the productivity of the more productive sector, yields similar results to the results presented above. Furthermore, even a neutral technical change, which is materialized by an equal increase in both sectors would yield the same results. Hence, extending our model to different types of technical change does not affect our results.

References

Acemoglu, Daron. 1998. "Why do new Technologies Complement Skills? Directed Technical Change and Wage Inequality." *The Quarterly Journal of Economics*, 113(4): 1055-1089.

Acemoglu, Daron, and Facrizio Zilibotti. 2001. "Productivity Differences." *Quarterly Journal of Economics*, 116(2), 563-606.

Acemoglu, Daron, and Veronica Guerrieri. 2008. “Capital Deepening and Non-Balanced Economic Growth”, *Journal of Political Economy,* 116(3): 467-498.

Acemoglu, Daron. 2009. “Introduction to Modern Economic Growth.” Princeton University Press.

Autor, David H. 2014. “Skills, Education, and the Rise of Earnings Inequality among the "Other 99 Percent".” *Science*, 344: 843-885.

Autor, David H., Katz, Lawrence F., and Melissa S. Kearney. 2008. “Trends in US Wage Inequality: Revising the Revisionists.” *The Review of Economics and Statistics*, 90(2): 300-323.

Baumol, William. 1967. “Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis”. *American Economic Review*, 57(3): 415-426.

Buera, Francisco J., and Joseph P. Kaboski. 2012. “The Rise of the Service Economy.” *American Economic Review*, 102(6): 2540-2569.

Buera, Francisco J., Kaboski, Joseph, and Richard Rogerson. 2015. “Skill Biased Structural Change.” *NBER Working Paper 21165*.

Caselli, Francesco, and Wilbur J. Coleman. 2006. “The World Technology Frontier.” *American Economic Review*, 96(3): 499-522.

Chusseau, Nathalie, Dumont, Michel and Joël Hellier. 2008 “Explaining Rising Inequality: Skill-biased Technical Change and North–South Trade.” *Journal of Economic Surveys*, 22(3): 409-457.

Cinar Baymul and Kunal Sen. 2020. “Was Kuznets Right? New Evidence on the Relationship between Structural Transformation and Inequality.” *The Journal of Development Studies*, 56(9): 1643-1662.

Comin, Diego A., Lashkari, Danial, and Martí Mestieri. 2015. “Structural Change with Long-Run Income and Price Effects.” No. w21595. National Bureau of Economic Research.

Diebolt, Claude, Le Chapelain, Charlotte, and Audrey Rose Menard. "Neither the elite, nor the mass. The rise of intermediate human capital during the French industrialization process." *Cliometrica* (forthcoming).

Diao, Xinshen., McMillan, Margaret, and Dani Rodrik. 2019. "The recent growth boom in developing economies: A structural change perspective." In Machiko Nissanke and José Antonio Ocampo (Eds), *The Palgrave Handbook of Development Economics*.

Duarte, Margarida, and Diego Restuccia. 2010. The Role of Structural Transformation in Aggregate Productivity. *The Quarterly Journal of Economics*, 125, 129–173.

Franck, Raphaël, and Oded Galor. "Flowers of Evil? Industrialization and Long Run Development." *Journal of Monetary Economics* (forthcoming).

Galor, Oded, and Omer Moav. 2000. "Ability-Biased Technological Transition, Wage Inequality, and Economic Growth." *The Quarterly Journal of Economics*, 115: 469-497.

Galor, Oded, and Daniel Tsiddon. 1997. "The Distribution of Human Capital and Economic Growth." *Journal of Economic Growth*, 2(1): 93-124.

Goldin, Claudia, and Lawrence F. Katz. 2007. "Long-Run Changes in the US Wage Structure: Narrowing, Widening, Polarizing." *NBER Working Paper 13568*.

Hall, Robert E., and Charles I. Jones. 1999. "Why do Some Countries Produce so much more Output Per Worker than Others?" *The Quarterly Journal of Economics*, 114(1): 83-116.

Haraguchi, N., Cheng, C. F. C., & Smeets, E. 2017. "The Importance of Manufacturing in Economic Development: Has this changed?" *World Development*, 93: 293–315.

Herrendorf, B., Rogerson, R., and Valentinyi, Á. 2014. "Growth and Structural Transformation." In P. Aghion & S. Durlauf (Eds.), *Handbook of economic growth* (855–941). Amsterdam: Elsevier.

Hutter, Christian, and Enzo Weber. 2017. "Labour Market Effects of Wage Inequality and Skill-Biased Technical Change in Germany." *IAB Discussion Paper No. 5*-2017.

Jiancai Pi, Pengqing Zhang. 2018. “Skill-Biased Technological Change and Wage Inequality in Developing Countries.” *International Review of Economics & Finance*, 56: 347-362.

Kongsamut, Piyabha, Rebelo, Sergio, and Danyang Xie. 2001. “Beyond Balanced Growth.” *Review of Economic Studies*, 68: 869-882.

Krusell. Per, Lee E. Ohanian, José-Víctor Ríos-Rull, and Giovanni L. Violante. 2000. “Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis.” *Econometrica*, 68(5): 1029-1053.

Maoz, Yishay D., and Omer Moav. 1999. “Intergenerational Mobility and the Process of Development.” *The Economic Journal*, 109: 677-697.

Nolan, Brian, Matteo G. Richiardi, and Luis Valenzuela. 2019 . “The drivers of income inequality in rich countries.” *Journal of Economic Surveys*, 33(4): 1285-1324.

Nuvolari, Alessandro., Bart Verspagen, and Nick von Tunzelmann. 2006. “The Diffusion of the Steam Engine in Eighteenth-Century Britain.” In: Andreas Pyka and Horst Hanusch (Eds.), *Applied Evolutionary Economics and the Knowledge-Based Economy*, Edward Elgar Publishers.

McMillan, Margaret., Rodrik, Dani., and Verduzco-Gallo, Íñigo. 2014. “Globalization, structural change, and productivity growth, with an update on Africa.” *World Development*, 63: 11–32.

Parente, Stephen L., Richard Rogerson, and Randall Wright. 2000. “Homework in Development Economics: Household Production and the Wealth of Nations.” *Journal of Political economy*, 108(4): 680-687.

Restuccia, Diego. 2004. “Barriers to Capital Accumulation and Aggregate Total Factor Productivity.” *International Economic Review*, 45(1): 225-238.

Restuccia, Diego and Richard Rogerson. 2013. “Misallocation and Productivity.” *Review of Economic Dynamics*, 16 (1): 1-10.

Romer, Paul. 1993. "Idea Gaps and Object Gaps in Economic Development." *Journal of Monetary Economics*, 32(3): 543-573.

Sauer, Petra, Narasimha, D. Rao, and Shonali Pachauri. 2015. "The Race Between Education and Technology Revisited." unpublished memo.

Święcki, Tomasz. 2017. "Determinants of Structural Change." *Review of Economic Dynamics*, 24: 95-131.

Trew, Alex.v. 2014. "Spatial Takeoff in the First Industrial Revolution." *Review of Economic Dynamics*, 17(4): 707-725.

Weil, David, 2013. "Economic Growth." Pearson Education Inc.

Zeira, Joseph. 2009. "Why and How Education Affects Economic Growth." *Review of International Economics*, 17(3): 602-614.

Appendix A

In this appendix we prove that the steady state level of H_t is unique. Based on (12), this steady state level is the level of H which serves as the root of the following equation:

(A.1) $$(1+\alpha)^2 \cdot \gamma^2 \cdot H^{2-\alpha} - \gamma \cdot \phi \cdot H + 2 \cdot \alpha \cdot (1+\alpha) \cdot \gamma \cdot H^{1-\alpha} - 2 \cdot \phi = 0$$

We define the LHS of (A.1) as the function $\psi(H)$ and decompose it to:

(A.2) $$\psi(H) \equiv x(H) + z(H)$$

where:

(A.3) $$x(H) \equiv (1+\alpha)^2 \cdot \gamma^2 \cdot H^{2-\alpha} - \gamma \cdot \phi \cdot H \ ,$$

(A.4) $$z(H) \equiv 2 \cdot \alpha \cdot (1+\alpha) \cdot \gamma \cdot H^{1-\alpha} - 2 \cdot \phi .$$

(A.3) immediately yields that $x(0) = 0$ and $\underset{H \to \infty}{Lim}\, x(H) = \infty$. It also follows from (A.3) that:

(A.5) $$x'(\overline{H}) = (1+\alpha)^2 \cdot \gamma^2 \cdot (2-\alpha) \cdot H^{1-\alpha} - \phi \cdot \gamma ,$$

which implies that $x'(0) < 0$ and $\underset{H \to \infty}{Lim}\, x'(H) = \infty$. It also follows from (A.5) that $x(H)$ has a single minimum point. Thus, $x(H)$ is a u-shaped function of *H* which falls from 0, at *H* = 0, to a negative minimum and from then on monotonically rises with *H*, reaches positive values and continues to infinity. From (A.3) it follows that $x(H)$ crosses the horizontal axis at

(A.6) $$H = \left[\frac{\phi}{\gamma \cdot (1+\alpha)^2} \right]^{\frac{1}{1-\alpha}} \equiv H_1 .$$

It is also immediate to see from (A.4) that $z(0) < 0$ and that $z'(H) > 0$ for all values of *H*. Applying H_1, as captured by (A.6) in (A.4) yields:

(A.7) $$z(H) = -4 \cdot \phi$$

From these properties of $x(H)$ and $z(H)$ it follows that $\psi(H)$ is a function which has negative values in the range [0, H_1] and from then on it monotonically rising in *H* reaching positive values and continuing to infinity. This implies that it crosses the horizontal axis only once and proves that the steady-state equation (A.1) has a single root.

Figure 3 presents the properties of $x(H)$, $z(H)$ and $\psi(H)$ described in this appendix.

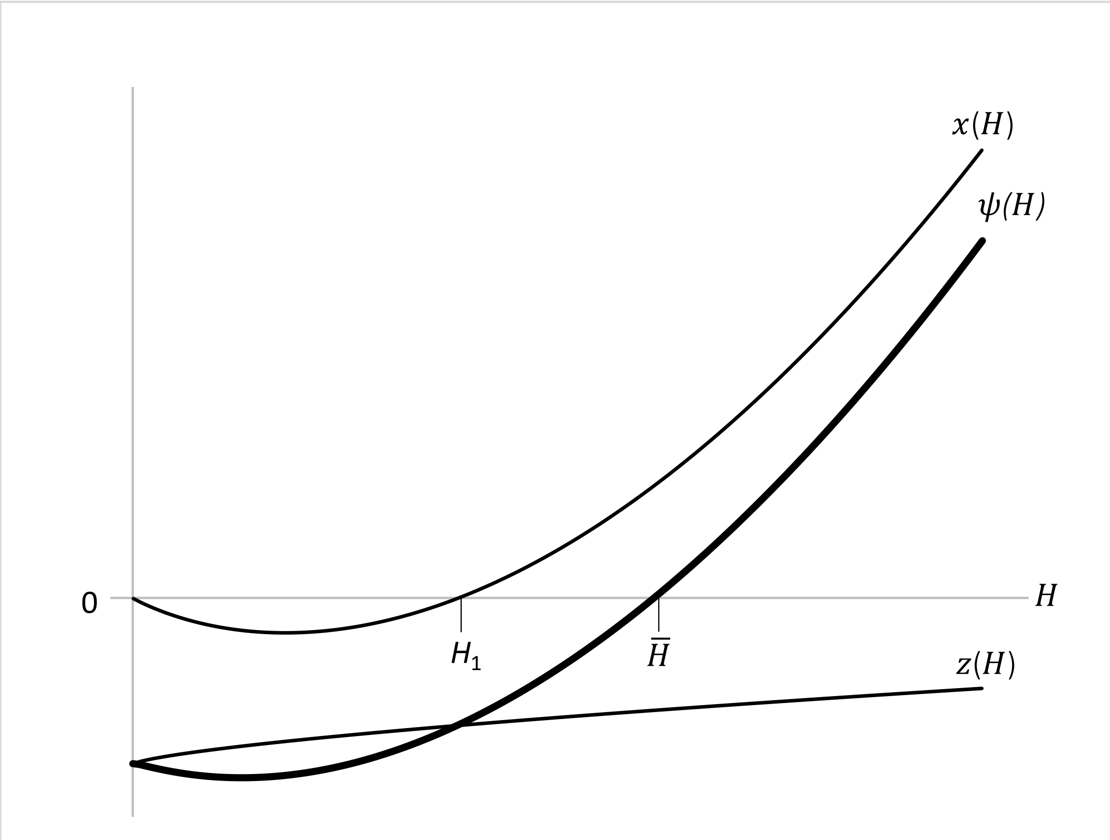

Figure 3: The uniqueness of the steady state level of *H*.

Since $\overline{H} > H_1$, equation (A.7) provides a sufficient condition for the economy to reach a steady state with $\overline{H}$ = 1:

$$A_H > \left(A_L^{\frac{1}{1-\alpha}} + M^{\frac{1}{1-\alpha}} \right)^{1-\alpha} > A_L, \tag{A.9}$$

where $M \equiv \frac{1+\alpha}{\alpha^{\alpha} \cdot (1-\alpha)^{2-\alpha} \cdot \beta} > 0$.

Thus, if A_H is sufficiently above A_L, then the economy reaches a steady state in which only the advanced sector is active. This is merely a sufficient condition, and as established numerically in sub-section 3.4 – a steady state with $\overline{H} < 1$ is possible too.